Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

$$=\frac{3a^2}{2}\log\sec\theta-2a^2\log\sec\theta+a^2\sin^2\theta\Big]_0^{\frac{1}{6}\pi}=a^2\sin^2\theta-\frac{a^2}{2}\log\sec\theta\Big]_0^{\frac{1}{6}\pi}$$

$$=\frac{3a^2}{4}-\frac{a^2}{2}\log 2.$$

[**Note.** **In the figure, the loop in the fourth quadrant should be in the first, and the one in the second in the third.**

92. Proposed by B. F. SINE, Principal of Capon Bridge Normal School, Capon Bridge, W. Va.

How much wood is taken from a log 12 inches in diameter, by boring a two-inch hole through the center, the axis of the hole being perpendicular to axis of log ?

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; J SCHEFFER, A. M., Hagerstown, Md., and WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.

$V=8\int\int\int dxdydz=$volume.

The equation of the surface of the cylinder corresponding to the log is

$$x^2+y^2=36=R^2,$$

and the equation of the surface of the cylinder corresponding to the auger-hole is

$$x^2+z^2=1=r^2.$$

$$V=8\int\int\int ydxdz=8\int_o^r\int_0^{\sqrt{(r^2-x^2)}}\sqrt{(R^2-x^2)}dxdy.$$

$$\therefore\ V=8\int_o^r\sqrt{[(r^2-x^2)(R^2-x^2)]}dx$$

$$=\tfrac{8}{3}R^3\{[1+(r/R)^2]E(r/R)-[1-(r/R)^2]F(r/R)\}$$

$$=576\{(1+\tfrac{1}{36})E(\tfrac{1}{6})-(1-\tfrac{1}{36})F(\tfrac{1}{6})\}$$

$$=576\{\tfrac{37}{36}E(\tfrac{1}{6})-\tfrac{35}{36}F(\tfrac{1}{6})\}\text{ cubic inches.}$$

Also $V=8\int_o^r\sqrt{(r^2-x^2)}\left[R-\frac{x^2}{2R}-\frac{x^4}{8R^3}-\ldots\ldots-\frac{1.3.5.7\ldots.(2n-1)x^{2n}}{2.4.6\ldots.2nR^{2n-1}}\right]dx.$

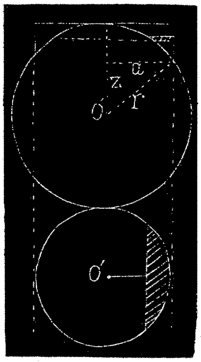
Now $\int_o^r\sqrt{(r^2-x^2)}x^{2n}dx=\frac{1.3.5.7\ldots.(2n-1)}{2.4.6.8\ldots.(2n+2)}\pi r^{n+2}.$

$$\therefore\ V=4r^2R\pi\left[\tfrac{1}{2}-\frac{r^2}{16R^2}-\frac{r^4}{128R^4}-\ldots\ldots-\frac{(1.3.5.7\ldots\ 2n-3)^2(2n-1)r^{2n}}{(2.4.6\ldots\ldots.\ 2n)^2(2n+2)R^{2n}}-\ldots\right]$$

$$\therefore\ V=24\pi(\tfrac{1}{2}-\tfrac{1}{576}-\tfrac{1}{165888}-\tfrac{5}{95551488}-\ldots\ldots-)=37.56784\text{ cubic inches.}$$

II. Solution by GEORGE R. DEAN, A. M., Professor of Mathematics, University of Missouri School of Mines and Metallurgy, Rolla, Mo.

This is easily seen to depend on the finding of the volume enclosed between the larger cylinder and a tangent plane, and the smaller cylinder.



Let r=radius of large cylinder.

a=radius of small cylinder.

z=distance of cutting plane from axis of the large cylinder.

We have then to evaluate the integrals

$$a^2\int_{\sqrt{(r^2-a^2)}}^{r}\cos^{-1}\frac{\sqrt{(r^2-z^2)}}{a}dz-\int_{\sqrt{(r^2-a^2)}}^{r}(r^2-z^2)\sqrt{(a^2-r^2+z^2)}dz.$$

Putting $\dfrac{\sqrt{(r^2-z^2)}}{a}=\sin\theta$, the first integral becomes

$$-\frac{\pi a^4}{2}\int\frac{\sin\theta\cos\theta d\theta}{\sqrt{(r^2-a^2\sin^2\theta)}}+a^4\int\frac{\theta\sin\theta\cos\theta d\theta}{\sqrt{(r^2-a^2\sin^2\theta)}}.$$

The second becomes $-a^4\displaystyle\int\frac{\sin^2\theta\cos^2\theta d\theta}{\sqrt{(r^2-a^2\sin^2\theta)}}$.

$$\int\frac{\sin\theta\cos\theta d\theta}{\sqrt{(r^2-a^2\sin^2\theta)}}=-\frac{\sqrt{(r^2-a^2\sin^2\theta)}}{a^2}.$$

Combining the second integral with the latter part of the first, we have

$$\int(\theta+\sin\theta\cos\theta)\frac{\sin\theta\cos\theta d\theta}{\sqrt{(r^2-a^2\sin^2\theta)}}$$

which integrated by parts gives

$$\frac{-(\theta+\sin\theta\cos\theta)\sqrt{(r^2-a^2\sin^2\theta)}}{a^2}+\int\frac{2\cos^2\theta\sqrt{(r^2-a^2\sin^2\theta)}}{a^2}d\theta.$$

The last integral

$$=2\int\frac{\sqrt{(r^2-a^2\sin^2\theta)}}{a^2}d\theta-2\int\sin\theta\frac{\sqrt{(r^2-a^2\sin^2\theta)}}{a^2}d\theta.$$

We have then

$$\tfrac{1}{2}\text{ Volume}=\left[\frac{\pi a^2}{2}\sqrt{(r^2-a^2\sin^2\theta)}-a^2(\theta+\sin\theta\cos\theta)\sqrt{(r^2-a^2\sin^2\theta)}\right]_0^{\frac{1}{2}\pi}$$

$$+2a^2\int\sqrt{(r^2-a^2\sin^2\theta)}d\theta-2a^2\int_0^{\frac{1}{2}\pi}\sin^2\theta\sqrt{(r^2-a^2\sin^2\theta)}d\theta.$$

When $\theta=0$, the first line is $\frac{1}{2}(\pi a^2r)$.
When $\theta=\frac{1}{2}\pi$, its value is 0.

To evaluate $\int\sqrt{(r^2-a^2\sin^2\theta)}d\theta$, put $a=re$, expand by binomial formula, and integrate the terms.

$$(1-e^2\sin^2\theta)^{\frac{1}{2}}=1-\tfrac{1}{2}e^2\sin^2\theta-\tfrac{1}{8}e^4\sin^4\theta-\tfrac{1}{16}e^6\sin^6\theta-\ldots\ldots$$

Then $\displaystyle\int_0^{\frac{1}{2}\pi}\sqrt{(r^2-a^2\sin^2\theta)}d\theta=\tfrac{1}{2}(\pi r)(1-\tfrac{1}{4}e^2-\tfrac{3}{64}e^4-\tfrac{5}{256}e^6\ldots\ldots)$

The remaining integral treated in the same way gives

$$\tfrac{1}{2}\pi r(\tfrac{1}{2}-\tfrac{3}{16}e^2-\tfrac{5}{128}e^4-\tfrac{35}{2048}e^6-\ldots\ldots$$

Finally, $\text{Volume}=\dfrac{\pi a^2r}{8}\left(e^2+\tfrac{1}{8}e^4+\dfrac{5e^6}{128}+\ \ldots.\ \right)$

In our example, $e=\frac{1}{6}$, $a=1$, $r=6$.

Hence, $V=\dfrac{9\pi}{2}\left(\dfrac{1}{36}+\dfrac{1}{8\times1296}+\dfrac{5}{128\times6^6}+\ldots\ldots=\tfrac{1}{8}\pi(1+\tfrac{1}{288}\right)$ nearly.

This subtracted from πa^2r and the result doubled gives the volume common to the two cylinders.

MECHANICS.

90. **Proposed by WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.**

Adopting the hypothesis that the planets were originally all one mass revolving about a fixed center and were formed by an explosion of this mass at some point in its path; prove that, if the law of nature were that force varies directly as the distance, the planets would all have collided again simultaneously, and find an expression for the time between the explosion and collision.

Solution by the PROPOSER.

Regarding the original mass as a particle, the pieces after explosion, no matter what their initial velocities or directions, would all move in concentric ellipses; and as their paths intersect in one point, viz., the position of the original mass at the moment of explosion, they must all have another point in common at the extremity of the common diameter through the first common point.

We have for the equations of motion for any piece